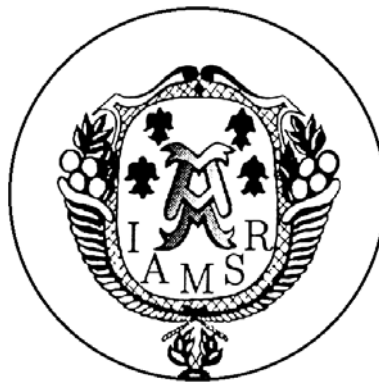


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Research Report 7/2008



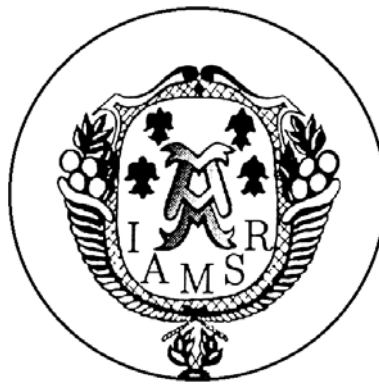
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Abstract

Real option analysis offers interesting insights on the value of assets and on the profitability of investments, which has made real options a growing field of academic research and practical application. Real option valuation is, however, often found to be difficult to understand and to implement due to the quite complex mathematics involved. Recent advances in modeling and analysis methods have made real option valuation easier to understand and to implement.

This paper presents a new method for real option valuation using fuzzy numbers that is based on findings from earlier real option valuation methods and from fuzzy real option valuation. The method is intuitive to understand and far less complicated than any previous real option valuation model to date.

Keywords: Real Option Valuation, Fuzzy Real Options, Fuzzy Numbers

1. Introduction

Real option valuation (ROV) is based on the observation that the possibilities financial options give their holder resemble the possibilities found in real investments, i.e., managerial flexibility, e.g., "an irreversible investment opportunity is much like a financial call option" (Pindyck, 1991). In other words, real option valuation is treating the different types of managerial flexibility as options and valuing managerial flexibility with option valuation models. Real options are useful both, as a mental model for strategic and operational decision-making, and as a valuation and numerical analysis tool. This paper concentrates on the use of real options in numerical analysis, and particularly on the derivation of real option value.

Real options are commonly valued with the same methods that have been used to value financial options, i.e., with Black-Scholes option pricing formula (Black & Scholes, 1973), with the binomial option valuation method (Cox, Ross, & Rubinstein, 1979), with Monte-Carlo based methods (Boyle, 1977), and with a number of later methods based on these. Most of the methods are complex and demand a good understanding of the underlying mathematics, issues that make their use difficult in practice. Recently, a novel approach to real option valuation was presented in (Mathews & Datar, 2007a), (Mathews & Salmon, 2007b), and in (Datar & Mathews, 2004), where the real option value is calculated from a pay-off distribution, derived from a probability distribution of the NPV for a project that is generated with a (monte-carlo) simulation. The

authors show that the results from the method converge to the results from the analytical Black-Scholes method. The method presented greatly simplifies the calculation of the real option value, making it more transparent and brings real option valuation as a method a big leap closer to practitioners.

All of the above mentioned models and methods use probability theory in their treatment of uncertainty, there are however, other ways than probability to treat uncertainty or imprecise future estimates, namely fuzzy logic and fuzzy sets. In classical set theory an element either (fully) belongs to a set or does not belong to a set at all. This type of bi-value, or true/false, logic is commonly used in financial applications. Bi-value logic, however, presents a problem, because financial decisions are generally made under uncertainty. Uncertainty means that it is impossible to give absolutely correct precise estimates of, e.g., future cash-flows. Fuzzy sets are sets that allow (have) gradation of belonging, such as "a future cash flow at year ten is about x euro". This means that fuzzy sets can be used to formalize inaccuracy that exists in human decision making and as a representation of vague, uncertain or imprecise knowledge, which human reasoning is especially adaptive to. "Fuzzy set-based methodologies blur the traditional line between qualitative and quantitative analysis, since the modeling may reflect more the type of information that is available rather than researchers' preferences" (Tarrazo, 1997) and indeed in economics "The use of fuzzy subsets theory leads to results that could not be obtained by classical methods." (Ponsard, 1988). The origins of fuzzy sets date back to an article by Lotfi Zadeh (Zadeh, 1965) where he developed an algebra for what he called fuzzy sets. This algebra was created to handle imprecise elements in our decision making processes, and is the formal body of theory that allows the treatment of practically all decisions in an uncertain environment. "Informally, a fuzzy set is a class of objects in which there is no sharp boundary between those objects that belong to the class and those that do not" (Bellman & Zadeh, 1970).

DEFINITION. Let $X = \{x\}$ denote a collection of objects (points) denoted generically by x . Then a fuzzy set A in X is a set of ordered pairs

$$Eq. 1. \quad A = \{(x, \mu_A(x))\}, \quad x \in X$$

where $\mu_A(x)$ is termed the grade of membership of x in A , and $\mu_A: X \rightarrow M$ is a function from X to a space M called the membership space. When M contains only two points, 0 and 1, A is non fuzzy and its membership function becomes identical with the characteristic function of a crisp set. This means that crisp sets are a subset of fuzzy sets. A fuzzy number is a normal, convex fuzzy set whose referential set is the real numbers $X \in R$.

Fuzzy set theory uses fuzzy numbers to quantify subjective fuzzy observations or estimates. Such subjective observations or estimates can be, e.g., estimates of future cash flows from an investment. To estimate future cash flows and discount rates "One usually employs educated guesses, based on expected values or other statistical techniques" (Buckley, 1987), which is consistent with the use of fuzzy numbers. In practical applications the most used fuzzy numbers are trapezoidal and triangular fuzzy numbers. They are used, because they make many operations possible and are intuitively understandable and interpretable.

When we replace non-fuzzy numbers (crisp, single) numbers that are commonly used in financial models with fuzzy numbers we can construct models that include the inaccuracy of human perception, or ability to forecast, within the (fuzzy) numbers. This makes these models more in line with reality, as they do not simplify uncertain distribution-like observations to a single point estimate that conveys the sensation of no-uncertainty. Replacing non-fuzzy numbers with fuzzy numbers means that the models that are built must also follow the rules of fuzzy arithmetic.

Fuzzy numbers (fuzzy logic) have been adopted to option valuation models in (binomial) pricing an option with a fuzzy payoff, e.g., in (Muzzioli & Torricelli, 2000), and in Black-Scholes valuation of financial options in, e.g., (Yoshida, 2003). There are also some option valuation models that present a combination of probability theory and fuzzy sets, e.g., (Zmeskal, 2001). Fuzzy numbers have also been applied to the valuation of real options in, e.g., (Carlsson & Fullér, 2003), (Collan, Carlsson, & Majlender, 2003), and (Carlsson & Majlender, 2005). There are also specific fuzzy models for the analysis of the value of optionality for very large industrial real investments, e.g., (Collan, 2004).

In the following section we will present a new method for valuation of real options from fuzzy numbers that is based on the previous literature on real option valuation, especially the findings presented in (Mathews et al., 2007a) and on fuzzy real option valuation methods, we continue by illustrating the method with a case and close with a discussion and conclusions.

2. New Method for Valuation of Real Options from Fuzzy Numbers

(Mathews et al., 2007b) and (Mathews et al., 2007a) present a practical probability theory based method for the calculation of real option value (ROV) and show that the method and results from the method are mathematically equivalent to the Black-Scholes formula (Black et al., 1973). The method is based on simulation generated probability distributions for the NPV of future project outcomes. The method implies that: “the real-option value can be understood simply as the average net profit appropriately discounted to Year 0, the date of the initial R&D investment decision, contingent on terminating the project if a loss is forecast at the future launch decision date.” The project outcome probability distributions are used to generate a payoff distribution, where the negative outcomes (subject to terminating the project) are truncated into one chunk that will cause a zero payoff, and where the probability weighted average value of the resulting payoff distribution is the real option value.

We use fuzzy numbers in representing the expected future distribution of possible project costs and revenues, and hence also the profitability (NPV) outcomes. When using fuzzy numbers the fuzzy NPV itself is a payoff distribution from the project.

The method presented in (Mathews et al., 2007a) implies that the weighted average of the positive outcomes of the payoff distribution is the real option value; in the case with fuzzy numbers the weighted average is the fuzzy mean value of the positive NPV outcomes (which is nothing more than the possibility weighted average). Derivation of the fuzzy mean value is presented in (Carlsson & Fullér, 2001).

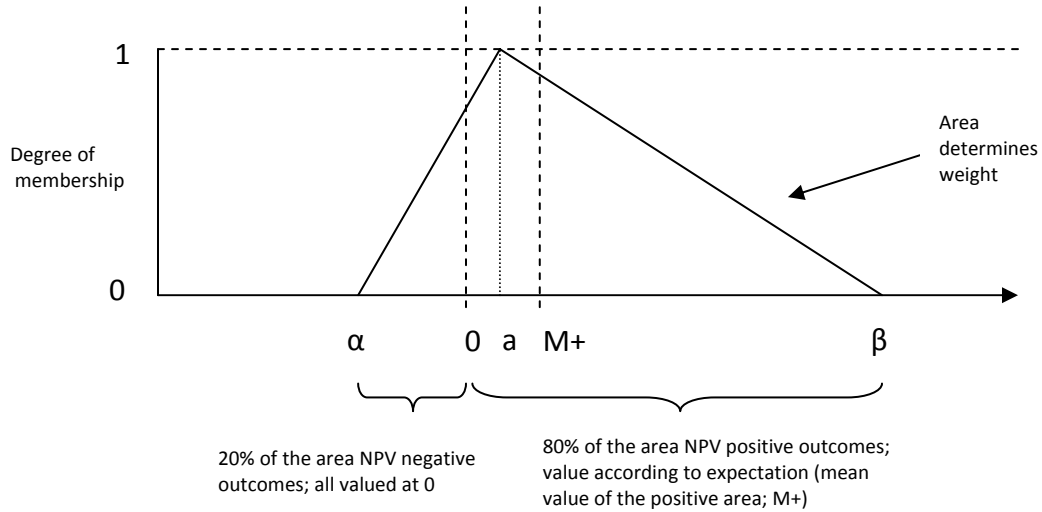


Figure 1. Triangular fuzzy number (a possibility distribution), defined by three points $[a, \alpha, \beta]$ describing the NPV of a prospective project.

This means that calculating the real option value (ROV) from a fuzzy NPV (distribution) is straightforward, it is the fuzzy mean of the possibility distribution with values below zero counted as zero, i.e., the area weighted average of the fuzzy mean of the positive values of the distribution and zero (for negative values).

Real option value is calculated from the fuzzy NPV as following:

$$\text{Eq. 2.} \quad \text{FROV} = \frac{A(\text{positive})}{A(\text{total})} \times \text{Fuzzy mean (positive NPV side)}$$

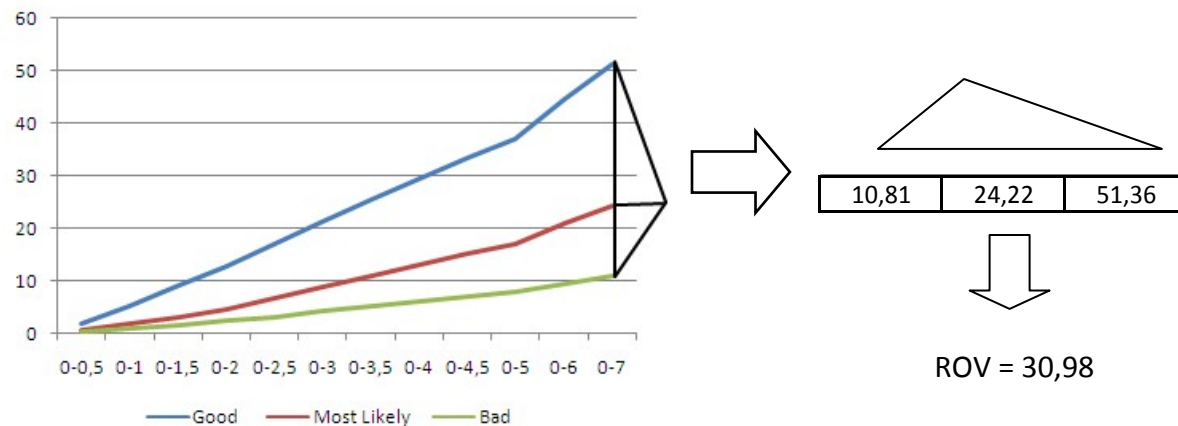
It is easy to see that when the whole fuzzy number is above zero ROV is the fuzzy mean of the number, and when the whole fuzzy number is below zero the ROV is 0.

The components of the new method are simply the observation that real option value is the probability weighted average of the positive values of a payoff distribution of a project, which is nothing more than the fuzzy NPV of the project, and that for fuzzy numbers the probability weighted average of the positive values of the payoff distribution is nothing more than the weighted fuzzy mean of the positive values of the fuzzy NPV, when we use fuzzy numbers.

3. Case: Using the New Method in Analyzing Acquisition Synergy as a Real Option

The problem at hand is to evaluate the value of uncertain synergies arising from a corporate acquisition that is estimated to last for seven years at maximum. The acquiring company has identified three possible scenarios, good, most likely, and bad, for the investments to realize the synergies and the synergy benefits. The scenario values are given by managers as non-fuzzy numbers, they can have used any type of analysis tools, or models to reach these scenarios. From these cost & benefit scenarios three scenarios for the NPV are combined (PV benefits – PV

investment costs), where the cost cash-flows (CF) are discounted at the risk free rate and the benefit CF discount rate is selected according to the risk (risk adjusted discount rate). The NPV is calculated for each of the three scenarios separately, i.e., good scenario costs are deducted



from good scenario benefits; this is not strictly in line with fuzzy arithmetic (as there the largest cost would be deducted from the smallest benefit to get the lower bound of the set).

Figure 2. Three NPV scenarios for the duration of the synergies that are used to generate (triangular) fuzzy NPV

The resulting fuzzy NPV is the payoff distribution for the synergies investment. The real option value for the investment can be calculated from the fuzzy NPV according to the formula presented in Eq. 2. In this case, as the whole distribution is above zero the ROV is nothing else than the fuzzy mean value of the fuzzy NPV.

The company managers are accustomed to giving information in the form of scenarios (usually three) and they have a set of methods for building the scenarios – usually coming from past experience and based on looking at issues like the most contributing single issues (or variables) and the markets. The scenario approach can be fully omitted and the future forecast can be done from the beginning with fuzzy numbers, the end result will be a fuzzy NPV in both cases.

4. Discussion and Conclusions

There is reason to expect that the simplicity of the presented method is an advantage over more complex methods. Using triangular and trapezoidal fuzzy numbers make very easy implementations possible with the most commonly used spreadsheet software; this opens avenues for real option valuation to find its way to more practitioners. The method is flexible as it can be used when the fuzzy NPV is generated from scenarios or as fuzzy numbers from the beginning of the analysis. Fuzzy NPV is a distribution of the possible values that can take place for NPV; this means that it is by definition perceived as impossible at the time of the assessment that values outside of the number can happen – this is in line with the situation that real option value is zero when all the values of the fuzzy NPV are lower than zero. If we compare this to the presented case, we can see that in practice it is often that managers are not interested to use the full distribution of possible outcomes, but rather want to limit their assessment to the most

possible alternatives (and leaving out the “tails” of the distribution). We think that the tails should be included in the real option analysis, because even remote possibilities should be taken into consideration.

The method brings forth an issue that has not gotten very much attention in academia, the dynamic nature of the assessment of investment profitability, i.e., the assessment changes when information changes. As cash flows taking place in the future come closer, information changes, and uncertainty is reduced this should be reflected in the fuzzy NPV, the more there is uncertainty the wider the distribution should be, and when uncertainty is reduced the width of the distribution should decrease. Only under full certainty should the distribution be represented by a single number, as the method uses fuzzy NPV there is a possibility to have the size of the distribution decrease with a lesser degree of uncertainty, this is an advantage vis-à-vis probability based methods.

The common decision rules for ROV analysis are applicable with the ROV derived with the presented method. We suggest that the single number NPV needed for comparison purposes is derived from the (same) fuzzy NPV by calculating the fuzzy mean value. This means that in cases when all the values of the fuzzy NPV are greater than zero the single number NPV equals ROV, which indicates immediate investment.

We feel that the presented new method opens possibilities for making simpler generic and modular real option valuation tools that will help construct real options analyses for systems of real options that are present in many types of investments.

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